# Complex Analysis

## Cauchy-Riemann and Laplace Equation

Given that: , where are functions of or .

Cauchy-Riemann equation

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If are differentiable and satisfy equation then:

* is **diﬀerentiable** at and
* is an **analytic function**

If is analytic in domain , then both and satisfy the Laplace equations:

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## Basic formulas

Conversion of complex number between rectangular form and polar form

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Complex number in exponential form

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Euler's formula

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Complex -th exponential

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Complex -th roots

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## Laurent Series

Laurent series

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Power series

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Consequences of power series

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| 1. |  |  |
| 2. |  |  |
| 3. |  |  |
| 4. |  |  |

# Laplace Transform

## Definition

If is continuous and there are positive numbers M, a such that, for all . Then is defined for all .

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## Properties

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## Formulas

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## Initial and Final Value Theorem

Initial-value theorem

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Final-value theorem

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## Heaviside – Unit step function

Given a piecewise-continuous function

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1. Express the piecewise-continuous function using the **unit step function**:

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2. Express the piecewise-continuous function using the **top hat function**:

## Convolution

Definition

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Solving a convolution: Find or

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| Let: |  |  |
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Taking inverse Laplace transform to find the result of the convolution

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# -transform

## Definition

Causal sequence:

Infinite sequence:

The -transform of an **infinite** **sequence** is defined whenever the sum exists and where is a complex variable

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The Z Transform of a **causal sequence**:

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Where: is the -Transform operator, : is a -Transform pair.

## Properties

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## Formulas

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## Initial and Final Value Theorem

Initial-value theorem

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Final-value theorem

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# Fourier Series

## Full Range Series

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Where:

Odd function: , and

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Even function: , and

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Parseval’s identity:

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## Half Range Series

### Half Range Sine Series:

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### Half Range Cosine Series:

## Frequently Used Formulas

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